Linear Algebra

System of linear equations

Def: A linear equation in the n variables $x_1, x_2, x_3, \dots, x_n$ is an equation that can be written in the form $a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n = b$

e.g. $5x + 2y = 3 \Rightarrow$ straight line in plane

$5x + 2y + 3z = 8 \Rightarrow$ plane in space

Determine whether the following equations linear or not

a) $\sqrt{2}x + \frac{\pi}{4}y - (\sin\frac{\pi}{5})z = 1$ Linear

b) $xy + 2z = 1$ non linear

c) $\sqrt{2x} + \frac{\pi}{4}y - \sin(\frac{\pi}{5}z) = 1$ non linear

How to solve a system of linear equations?

a) $x - y = 1$
$x + y = 3$

$2x = 4 \quad x = 2 \quad y = 1$

b) $x - y = 2$
$2x - 2y = 4$

b) infinite no. of values because both lines are the same

c) $x - y = 1$
$x - y = 3$

c) no solution because both lines are parallel (same slope)

* A system of linear equations with real coefficients has either:-

a) a unique solution [consistent system]

b) infinite number of solutions [consistent system]

c) no solution [inconsistent system]

Ex. Solve the system

$x - y - z = 2$
$y + 3z = 5$
$5z = 10$

$z = \frac{10}{5} = 2$

$y + 3(2) = 5 \quad y = -1$

$x - (-1) - 2 = 2$

$x = 3$

Back substitution method

Matrices and Echelon Form

Matrix: It's a rectangular array of numbers with (m) rows and (n) columns

eg $\begin{pmatrix} 5 & 3 & 8 \\ 4 & 1 & 2 \\ 9 & 8 & 7 \\ 0 & 5 & 6 \end{pmatrix}$ $\Rightarrow$ 4 x 3 matrix

↓ ↓

rows x columns

Coefficient matrix: A matrix containing the coefficients of the variables in a system of equations

eg) $5x+3y-z=8$
$x+2y+3z=12$ $\Rightarrow$ $\begin{pmatrix} 5 & 3 & -1 \\ 1 & 2 & 3 \\ 4 & -3 & 8 \end{pmatrix}$
$4x-3y+8z=-25$

Augmented matrix: A matrix containing all coefficients in a system of equations

eg) $5x+3y-z=8$
$x+2y+3z=12$
$4x-3y+8z=-25$
$\begin{pmatrix} 5 & 3 & -1 & 8 \\ 1 & 2 & 3 & 12 \\ 4 & -3 & 8 & -25 \end{pmatrix}$

Echelon form: A matrix in row echelon form if it satisfies the following conditions:

1) Any rows consisting entirely of zeros are at the bottom of the matrix
2) In each non zero row, the first non zero entry is in a column to the left of any leading entries below it or the first non zero entry must have all entries below it are zeros

eg) $\begin{pmatrix} 2 & 4 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 4 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 & 5 \\ 0 & 6 & 9 \\ 0 & 0 & 6 \end{pmatrix}$

Both conditions are satisfied ∴ echelon form

first condition is only satisfied ∴ not echelon form

Second condition is only satisfied ∴ not echelon form